FREE VIBRATIONS OF THIN ELASTIC ORTHOTROPIC CYLINDRICAL PANEL WITH RIGID – CLAMPED EDGE GENERATOR

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ABSTRACT

Using the system of equations corresponding to the classical theory of orthotropic cylindrical shells, the free vibrations of thin elastic orthotropic cylindrical panel with rigid – clamped edge generator are investigated. In order to calculate the natural frequencies and to identify the respective natural modes, the generalized Kantorovich-Vlasov method of reduction to ordinary differential equations is used. Dispersion equations for finding the natural frequencies of possible types of vibrations are derived. An asymptotic relation between the dispersion equations of the problem at hand and the analogous problem for a cantilever rectangular plate is established. An algorithm for separating possible boundary vibrations is presented. As an example, the values of dimensionless characteristics of natural frequencies are derived for an orthotropic cylindrical panel.

Key words: *free vibrations, eigenfrequencies, cylindrical panel, eigenfunctions.*

INTRODUCTION

It is known that, at the free edge of an orthotropic plate planar and flexural vibrations can occur independently of each other (Norris A.N. 1994) –(Gulgazaryan, G.R., Gulgazaryan R.G., Khachanyan A.A. 2013). When the plate is bent these vibrations become coupled and giving raise to two new types of vibrations localized at the free edge: predominantly tangential and predominantly bending vibrations. The transformation of the one type of vibration into the other occurs at the free edge of a thin cylindrical elastic panel. For these vibrations a complex distribution of frequencies of natural vibrations occurs depending on the geometrical and mechanical parameters of finite and infinite cylindrical panels (Gulgazaryan, G.R., Gulgazaryan R.G., Khachanyan A.A. 2013)-(Ghulghazaryan G.R., Ghulghazaryan L.G. 2020). With the increase of the number of free edges of a cylindrical panel the distribution becomes increasingly complex (Gulgazaryan, G.R., Gulgazaryan L.G., Saakyan R.D. 2008) --(Ghulghazaryan G.R., Ghulghazaryan L.G. 2020). Therefore, the investigation of the edge resonance of cantilever plates and cylindrical panels with rigid – clamped edge generator is one of the most difficult problems in the theory of vibrations of plates and shells (Vilde M.V., Kaplunov Yu.D., Kassovich L.Yu. 2010). These difficulties are resolved by using a combination of analytical and asymptotic theories, as well as by numerical methods.

In the present work, for the first time, free vibrations of a cantilever plate and a cylindrical panel with rigid – clamped edge generator are investigated. Such elements are important components of modern structures and constructions. Therefore, the question of free vibrations of these elements is of vital importance and it demands special attention. It is proved that the problem prevents separation of variables for the given boundary conditions. It can be proved that such problems for cylindrical shells of orthotropic materials with simple boundary conditions are self-conjugate and nonnegative definite. Therefore, the generalized Kantorovich-Vlasov method can be applied to them

(Vlasov V.Z. 1932)-(Mikhlin S.G. 1970). As the basic functions the following eigenfunctions of the problem are used:

$$w^{VIII} = \theta^8 w,\ w|_{\alpha=0,l} = w'|_{\alpha=0,l} = w''|_{\alpha=0,l} = w'''|_{\alpha=0,l} = 0,\ \ 0 \le \alpha \le l\,. \tag{1}$$

The problem (1) is a self-conjugate and has a positive simple discrete spectrum with a limit point at the infinity. The eigenfunctions corresponding to the eigenvalues θ_m^8, $m = \overline{1,\infty}$, of the problem (1) have the form:

$$w_m(\theta_m\alpha) = \frac{\Delta_1}{\Delta}x_1(\theta_m\alpha) + \frac{\Delta_2}{\Delta}x_2(\theta_m\alpha) + \frac{\Delta_3}{\Delta}x_3(\theta_m\alpha) + x_4(\theta_m\alpha)\,, 0 \le \alpha \le l, m = \overline{1,\infty} \tag{2}$$

$$x_1(\theta_m\alpha) = ch\theta_m\alpha - ch\frac{\theta_m\alpha}{\sqrt{2}}\cos\frac{\theta_m\alpha}{\sqrt{2}} - sh\frac{\theta_m\alpha}{\sqrt{2}}\sin\frac{\theta_m\alpha}{\sqrt{2}},$$

$$x_2(\theta_m\alpha) = sh\theta_m\alpha - \sqrt{2}ch\frac{\theta_m\alpha}{\sqrt{2}}\sin\frac{\theta_m\alpha}{\sqrt{2}},\ x_3(\theta_m\alpha) = sh\theta_m\alpha - \sqrt{2}sh\frac{\theta_m\alpha}{\sqrt{2}}cos\frac{\theta_m\alpha}{\sqrt{2}},$$

$$x_4(\theta_m\alpha) = cos\theta_m\alpha - ch\frac{\theta_m\alpha}{\sqrt{2}}\cos\frac{\theta_m\alpha}{\sqrt{2}} + sh\frac{\theta_m\alpha}{\sqrt{2}}\sin\frac{\theta_m\alpha}{\sqrt{2}}$$

$$\Delta = \begin{vmatrix} x_1(\theta_m l)x_2(\theta_m l)x_3(\theta_m l) \\ x_1'(\theta_m l)x_2'(\theta_m l)x_3'(\theta_m l) \\ x_1''(\theta_m l)x_2''(\theta_m l)x_3''(\theta_m l) \end{vmatrix},\quad \Delta_1 = -\begin{vmatrix} x_4(\theta_m l)x_2(\theta_m l)x_3(\theta_m l) \\ x_4'(\theta_m l)x_2'(\theta_m l)x_3'(\theta_m l) \\ x_4''(\theta_m l)x_2''(\theta_m l)x_3''(\theta_m l) \end{vmatrix} \tag{3}$$

$$\Delta_2 = -\begin{vmatrix} x_1(\theta_m l)x_4(\theta_m l)x_3(\theta_m l) \\ x_1'(\theta_m l)x_4'(\theta_m l)x_3'(\theta_m l) \\ x_1''(\theta_m l)x_4''(\theta_m l)x_3''(\theta_m l) \end{vmatrix},\ \Delta_3 = -\begin{vmatrix} x_1(\theta_m l)x_2(\theta_m l)x_4(\theta_m l) \\ x_1'(\theta_m l)x_2'(\theta_m l)x_4'(\theta_m l) \\ x_1''(\theta_m l)x_2''(\theta_m l)x_4''(\theta_m l) \end{vmatrix}$$

This eigenfunctions with their first and second derivatives define an orthogonal basis in a Hilbert space $L_2[0,s]$ (Mikhlin S.G. 1970). Here θ_m, $m = \overline{1,\infty}$, are the positive zeros of the determinant of Vronsky for functions (3) at the point $\alpha = l$. Let us define

$$\beta_m' = \frac{\int_0^l \left(w_m'(\theta_m\alpha)\right)^2 d\alpha}{\int_0^l \left(w_m(\theta_m\alpha)\right)^2 d\alpha},\ \beta_m'' = \frac{\int_0^l \left(w_m''(\theta_m\alpha)\right)^2 d\alpha}{\int_0^l \left(w_m'(\theta_m\alpha)\right)^2 d\alpha} \tag{4}$$

Notice that, the derivatives in formulas (3) and (4) are taken with respect to $\theta_m\alpha$ and $\beta_m' \to 1$, $\beta_m'' \to 1$ at $m \to +\infty$.

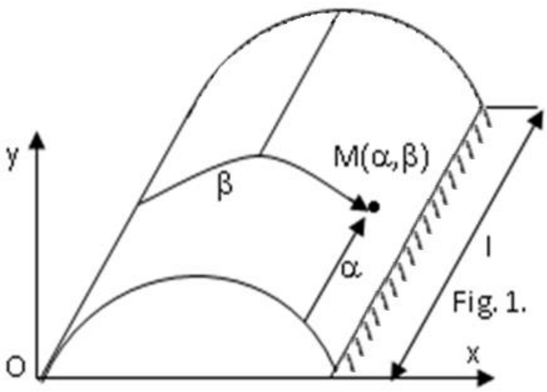
THE STATEMENT OF THE PROBLEM AND THE BASIC EQUATIONS

It is assumed that the generatrices of the cylindrical panel are orthogonal to the ends of the panel. The curvilinear coordinates (α,β) are defined on the median surface of the shell where $\alpha(0 \le \alpha \le l)$ and $\beta(0 \le \beta \le s)$ are the lengths of the generatrix and the directing circumference, respectively; l – is the length of the panel;and s – is the length of the directing circumference.

As the initial equations describing vibrations of the panel, we will use the equations corresponding to the classical theory of orthotropic cylindrical shells written in the selected curvilinear coordinates α and β (Fig. 1) (Ambartsumyan S.A. 1974):

$$
\begin{aligned}
& -B_{11}\frac{\partial^2 u_1}{\partial\alpha^2} - B_{66}\frac{\partial^2 u_1}{\partial\beta^2} - (B_{12}+B_{66})\frac{\partial^2 u_2}{\partial\alpha\partial\beta} + \frac{B_{12}}{R}\frac{\partial u_3}{\partial\alpha} = \lambda u_1, \\
& -(B_{12}+B_{66})\frac{\partial^2 u_1}{\partial\alpha\partial\beta} - B_{66}\frac{\partial^2 u_2}{\partial\alpha^2} - B_{22}\frac{\partial^2 u_2}{\partial\beta^2} + \frac{B_{22}}{R}\frac{\partial u_3}{\partial\beta} - \frac{\mu^4}{R^2}\left(4B_{66}\frac{\partial^2 u_2}{\partial\alpha^2} + \right. \\
& \left. + B_{22}\frac{\partial^2 u_2}{\partial\beta^2}\right) - \frac{\mu^4}{R}\left(B_{22}\frac{\partial^3 u_3}{\partial\beta^3} + (B_{12}+4B_{66})\frac{\partial^3 u_3}{\partial\beta\partial\alpha^2}\right) = \lambda u_2, \\
& \mu^4\left(B_{11}\frac{\partial^4 u_3}{\partial\alpha^4} + 2(B_{12}+2B_{66})\frac{\partial^4 u_3}{\partial\alpha^2\partial\beta^2} + B_{22}\frac{\partial^4 u_3}{\partial\beta^4}\right) + \frac{\mu^4}{R}\left(B_{22}\frac{\partial^3 u_2}{\partial\beta^3} + \right. \\
& \left. + (B_{12}+4B_{66})\frac{\partial^3 u_2}{\partial\beta\partial\alpha^2}\right) - \frac{B_{12}}{R}\frac{\partial u_1}{\partial\alpha} - \frac{B_{22}}{R}\frac{\partial u_2}{\partial\beta} + \frac{B_{22}}{R^2}u_3 = \lambda u_3
\end{aligned}
\tag{1.1}
$$

Here, u_1, u_2 and u_3 are projections of the displacement vector on the directions α and β, and on the normal to the median surface of the panel, respectively; R is the radius of the directing circumference of the median surface; $\mu^4 = h^2/12$ (h is the thickness of the panel);



$\lambda = \omega^2\rho$, where ω is the angular frequency, ρ is the density of the material; B_{ij} are the elasticity coefficients. The boundary conditions has the form **(Ambartsumyan S.A. 1974):**

$$
\begin{aligned}
& \left.\frac{\partial u_1}{\partial\alpha} + \frac{B_{12}}{B_{11}}\left(\frac{\partial u_2}{\partial\beta} - \frac{u_3}{R}\right)\right|_{\alpha=0,l} = 0, \qquad \left.\frac{\partial u_2}{\partial\alpha} + \frac{\partial u_1}{\partial\beta} + \frac{4\mu^4}{R}\left(\frac{\partial^2 u_3}{\partial\alpha\partial\beta} + \frac{1}{R}\frac{\partial u_2}{\partial\alpha}\right)\right|_{\alpha=0,l} = 0, \\
& \left.\frac{\partial^2 u_3}{\partial\alpha^2} + \frac{B_{12}}{B_{11}}\left(\frac{\partial^2 u_3}{\partial\beta^2} + \frac{1}{R}\frac{\partial u_2}{\partial\beta}\right)\right|_{\alpha=0,l} = 0, \quad \left.\frac{\partial^3 u_3}{\partial\alpha^3} + \frac{B_{12}+4B_{66}}{B_{11}}\left(\frac{\partial^3 u_3}{\partial\alpha\partial\beta^2} + \frac{1}{R}\frac{\partial^2 u_2}{\partial\alpha\partial\beta}\right)\right|_{\alpha=0,l} = 0.
\end{aligned}
\tag{1.2}
$$

$$
\begin{aligned}
& \left.\frac{B_{12}}{B_{22}}\frac{\partial u_1}{\partial\alpha} + \frac{\partial u_2}{\partial\beta} - \frac{u_3}{R}\right|_{\beta=0} = 0, \quad \left.\frac{B_{12}}{B_{22}}\frac{\partial^2 u_3}{\partial\alpha^2} + \frac{\partial^2 u_3}{\partial\beta^2} + \frac{1}{R}\frac{\partial u_2}{\partial\beta}\right|_{\beta=0} = 0, \\
& \left.\frac{\partial^3 u_3}{\partial\beta^3} + \frac{B_{12}+4B_{66}}{B_{22}}\frac{\partial^3 u_3}{\partial\beta\partial\alpha^2} + \frac{1}{R}\frac{\partial^2 u_2}{\partial\beta^2} + \frac{4B_{66}}{B_{22}}\frac{1}{R}\frac{\partial^3 u_2}{\partial\alpha^2}\right|_{\beta=0} = 0, \quad \left.\frac{\partial u_2}{\partial\alpha} + \frac{\partial u_1}{\partial\beta}\right|_{\beta=0} = 0.
\end{aligned}
\tag{1.3}
$$

$$u_1\big|_{\beta=s}=u_2\big|_{\beta=s}=u_3\big|_{\beta=s}=\frac{\partial u_3}{\partial\beta}\bigg|_{\beta=s}=0 \tag{1.4}$$

Relations (1.2) and (1.3) are the conditions of free edges for $\alpha=0,l$ and $\beta=0$, respectively, while conditions (1,4) indicate that the edge generator $\beta=s$ is rigid-clamped.

THE DERIVATION AND ANALYSIS OF THE CHARACTERISTIC EQUATIONS

Let's formally replace the spectral parameter λ by λ_1,λ_2 and λ_3 in the first, second, and third equations of the system (1.1), respectively. The solution of system (1.1) is searched in the form

$$(u_1,u_2,u_3)=\{u_m w_m'(\theta_m\alpha),v_m w_m(\theta_m\alpha),w_m(\theta_m\alpha)\}\exp(\theta_m\chi\beta),\quad m=\overline{1,+\infty}. \tag{2.1}$$

Here, $w_m(\theta_m\alpha), m=\overline{1,\infty}$, are determined from (2) and u_m,v_m,χ are unknown constants. In this case, the conditions (1.2) are obeyed automatically. Let us insert Eq. (2.1) into Eq. (1.1). The obtained equations are multiplied by the vector functions $\left(w_m'(\theta_m\alpha),w_m(\theta_m\alpha),w_m(\theta_m\alpha)\right)$ in a scalar way and then integrated in the limits from 0 to l. From the first two equations we have

$$(c_m+\varepsilon_m^2a^2g_md_m)u_m=\varepsilon_m\left\{a_m+\frac{B_{22}(B_{12}+B_{66})}{B_{11}B_{66}}a^2\chi^2l_m+\frac{B_{22}B_{12}}{B_{11}B_{66}}\varepsilon_m^2a^2d_m\right\}, \tag{2.2}$$

$$(c_m+\varepsilon_m^2a^2g_md_m)v_m=\varepsilon_m\chi\left\{b_m-a^2g_ml_m\right\}. \tag{2.3}$$

From the third equation, by taking into account the relations (2.2) and (2.3), the characteristic equation is obtained

$$R_{mm}c_m+\varepsilon_m^2\left\{c_m-b_m\chi^2-\frac{B_{12}}{B_{22}}\beta_m'a_m+a^2[R_{mm}g_md_m+2l_mb_m\chi^2]+\right.$$
$$\left.\varepsilon_m^2a^2d_m(b_m+\frac{B_{12}}{B_{11}}\beta_m')-a^4g_ml_m^2\chi^2\right\}=0,\ m=\overline{1,\infty}. \tag{2.4}$$

$$a_m=-\left(\frac{B_{22}}{B_{11}}\chi^2-\frac{B_{12}}{B_{11}}\left(\eta_{2m}^2-\beta_m'\right)\right);\ b_m=\frac{B_{22}}{B_{11}}\left(\chi^2+\eta_{1m}^2\right)-B_1;\ d_m=\chi^2-\frac{4B_{66}}{B_{22}}\beta_m';$$

$$c_m=\frac{B_{22}}{B_{11}}\chi^4-B_2\chi^2+\left(\frac{B_{22}}{B_{11}}\eta_{1m}^2+\frac{B_{66}}{B_{11}}\eta_{2m}^2\right)\chi^2+(\beta_m'-\eta_{2m}^2)\left(\beta_m''-\frac{B_{66}}{B_{11}}\eta_{1m}^2\right); \tag{2.5}$$

$$B_1=\frac{B_{11}B_{22}\beta''-B_{12}^2\beta'_m-B_{12}B_{66}\beta'_m}{B_{11}B_{66}};\; B_2=\frac{B_{11}B_{22}\beta''_m-B_{12}^2\beta'_m-2B_{12}B_{66}\beta'_m}{B_{11}B_{66}};$$

$$g_m=\frac{B_{22}}{B_{11}}\chi^2-\frac{B_{22}}{B_{66}}\beta''_m+\frac{B_{22}}{B_{11}}\eta_{1m}^2;\; l_m=\chi^2-\frac{B_{12}+4B_{66}}{B_{22}}\beta'_m;\; \eta_{im}^2=\frac{\lambda_i}{B_{66}\theta_m^2},\; i=\overline{1,3};$$

$$R_{mm}=a^2\left(\chi^4-\frac{2(B_{12}+2B_{66})}{B_{22}}\beta'_m\chi^2+\frac{B_{11}}{B_{22}}\beta'_m\beta''_m\right)-\frac{B_{66}}{B_{22}}\eta_{3m}^2;\; a^2=\frac{h^2}{12}\theta_m^2;\; \varepsilon_m=\frac{1}{R\theta_m}.$$

Let $\chi_j,\ j=\overline{1,4}$, be pairwise different roots of Eq.(2.4) with non-positive real parts and $\chi_{4+j}=-\chi_j, j=\overline{1,4}$. Let $(u_1^{(j)},u_2^{(j)},u_3^{(j)}),\ j=\overline{1,8}$, be nontrivial solutions of type (2.1) of the system (1.1) at $\chi=\chi_j, j=\overline{1,8}$, respectively. The solution of the problem (1.1)-(1.4) is searched in the form

$$u_i=\sum_{j=1}^{8}u_i^{(j)}w_j,\; i=\overline{1,3}. \tag{2.6}$$

Let us insert Eq. (2.6) into the boundary conditions (1.3) and (1.4). Each of the obtained equation is multiplied by $w(\theta_m\alpha)$ or by $w'(\theta_m\alpha)$, respectively, and then integrated in the limits from 0 to l. As a result, we obtain the system of equations

$$\left.\begin{aligned}&\sum_{j=1}^{8}\frac{M_{ij}^{(m)}w_j}{c_m^{(j)}+\varepsilon_m^2a^2g_m^{(j)}d_m^{(j)}}=0,\quad i=\overline{1,4}\\ &\sum_{j=1}^{8}\frac{M_{ij}^{(m)}\exp(\theta_m\chi_j\alpha)w_j}{c_m^{(j)}+\varepsilon_m^2a^2g_m^{(j)}d_m^{(j)}}=0,\quad i=\overline{5,8}\end{aligned}\right.,\quad m=\overline{1,\infty} \tag{2.7}$$

$$M_{1j}^{(m)}=\chi_j^2b_m^{(j)}-\frac{B_{12}}{B_{22}}a_m^{(j)}\beta'_m-c_m^{(j)}-a^2l_m^{(j)}\chi_j^2b_m^{(j)}-\varepsilon_m^2a^2d_m^{(j)}\left(b_m^{(j)}-\frac{B_{12}}{B_{22}}\beta'_m\right),$$

$$M_{2j}^{(m)}=\chi_j\left\{a_m^{(j)}+b_m^{(j)}+a^2l_m^{(j)}\left(\frac{B_{12}B_{22}}{B_{11}B_{66}}\chi_j^2+\frac{B_{22}}{B_{66}}\beta''_m-\frac{B_{22}}{B_{11}}\eta_{1m}^2\right)+\frac{B_{12}B_{22}}{B_{11}B_{66}}a^2\varepsilon_m^2d_m^{(j)}\right\}, \tag{2.8}$$

$$M_{3j}^{(m)}=\left(\chi_j^2-\frac{B_{12}}{B_{22}}\beta'_m\right)c_m^{(j)}+\varepsilon_m^2\left(b_m^{(j)}\chi_j^2+a^2\frac{4B_{12}B_{66}}{(B_{22})^2}g_m^{(j)}(\beta'_m)^2\right),$$

$$M_{4j}^{(m)}=\chi_j\left(l_m^{(j)}c_m^{(j)}+\varepsilon_m^2d_m^{(j)}b_m^{(j)}\right),$$

$$M_{5j}^{(m)}=a_m^{(j)}+\frac{B_{22}(B_{12}+B_{66})}{B_{11}B_{66}}a^2\chi_j^2l_m^{(j)}+\frac{B_{22}B_{12}}{B_{11}B_{66}}\varepsilon_m^2a^2d_m^{(j)},$$

$$M_{6j}^{(m)}=\chi_j\left(b_m^{(j)}-a^2g_m^{(j)}l_m^{(j)}\right),$$

$$M_{7j}^{(m)}=c_m^{(j)}+\varepsilon_m^2a^2g_m^{(j)}d_m^{(j)},\; M_{8j}^{(m)}=\chi_j\left(c_m^{(j)}+\varepsilon_m^2a^2g_m^{(j)}d_m^{(j)}\right),\; j=\overline{1,8}.$$

The superscript j in parentheses means that the corresponding function is taken at $\chi = \chi_j$. In order to the system (2.7) has a nontrivial solution, it is necessary and sufficient that

$$\Delta = \exp\left(-\sum_{j=1}^{4} z_j\right) Det\left\| T_{ij}^{(m)} \right\|_{i,j=1}^{2} = 0, \quad m = \overline{1,\infty}, \tag{2.9}$$

$$\begin{aligned} &T_{11} = \left\| M_{ij}^{(m)} \right\|_{i,j=1}^{4}, \quad T_{12} = \left\| (-1)^{i-1} M_{ij}^{(m)} \exp(z_j) \right\|_{i,j=1}^{4}, z_j = \theta_m \chi_j s, \\ &T_{21} = \left\| M_{ij}^{(m)} \exp(z_j) \right\|_{i=5,j=1}^{8,4}, \quad T_{22} = \left\| (-1)^{i-1} M_{ij}^{(m)} \right\|_{i=5,j=1}^{8,4}. \end{aligned} \tag{2.10}$$

It is shown numerically that the left side of this equality becomes small when any two roots of Eq. (2.4) become close to each other. This highly complicates calculations and can lead to false solutions. It turns out that from the left side of Eq. (2.9) a multiplier that tends to zero can be separated when the roots approach each other. Let us introduce the following notations:

$$\begin{aligned} &[z_i z_j] = \theta_m s(\exp(z_i) - \exp(z_j))/(z_i - z_j), \; [z_i z_j z_k] = \theta_m s([z_i z_j] - [z_i z_k])/(z_j - z_k), \\ &[z_1 z_2 z_3 z_4] = \theta_m s([z_1 z_2 z_3] - [z_1 z_2 z_4])/(z_3 - z_4), \\ &\sigma_1 = \sigma_1(\chi_1, \chi_2, \chi_3, \chi_4) = \chi_1 + \chi_2 + \chi_3 + \chi_4, \\ &\sigma_2 = \sigma_2(\chi_1, \chi_2, \chi_3, \chi_4) = \chi_1\chi_2 + \chi_1\chi_3 + \chi_1\chi_4 + \chi_2\chi_3 + \chi_2\chi_4 + \chi_3\chi_4, \\ &\sigma_3 = \sigma_3(\chi_1, \chi_2, \chi_3, \chi_4) = \chi_1\chi_2\chi_3 + \chi_1\chi_2\chi_4 + \chi_1\chi_3\chi_4 + \chi_2\chi_3\chi_4, \\ &\sigma_4 = \sigma_4(\chi_1, \chi_2, \chi_3, \chi_4) = \chi_1\chi_2\chi_3\chi_4, \\ &\bar{\sigma}_k = \sigma_k(\chi_1, \chi_2, \chi_3, 0), \; \bar{\bar{\sigma}}_k = \sigma_k(\chi_1, \chi_2, 0, 0), \; k = \overline{1,4}. \end{aligned} \tag{2.11}$$

In this case, $\bar{\sigma}_4 = \bar{\bar{\sigma}}_4 = \bar{\bar{\sigma}}_3 = 0$. Let f_n, $n = \overline{1,6}$, be a symmetric polynomial of nth order in variables $\chi_1, \chi_2, \chi_3, \chi_4$. It is known that it can be uniquely expressed in terms of elementary symmetric polynomials. By introducing the notations

$$f_n = f_n(\sigma_1, \sigma_2, \sigma_3, \sigma_4); \; \bar{f}_n = f_n(\bar{\sigma}_1, \bar{\sigma}_2, \bar{\sigma}_3, 0); \; \bar{\bar{f}}_n = f_n(\bar{\bar{\sigma}}_1, \bar{\bar{\sigma}}_2, 0, 0), \; n = \overline{1,6}; \tag{2.12}$$

$$\begin{aligned} &f_1 = \sigma_1, \; f_2 = \sigma_1^2 - \sigma_2; \; f_3 = \sigma_1^3 - 2\sigma_1\sigma_2 + \sigma_3; \; f_4 = \sigma_1^4 - 3\sigma_1^2\sigma_2 + \sigma_2^2 + 2\sigma_1\sigma_3 - \sigma_4; \\ &\bar{f}_5 = \bar{\sigma}_1^5 - 4\bar{\sigma}_1^3\bar{\sigma}_2 + 3\bar{\sigma}_1\bar{\sigma}_2^2 + 3\bar{\sigma}_1^2\bar{\sigma}_3 - 2\bar{\sigma}_2\bar{\sigma}_3; \; \bar{\bar{f}}_6 = \bar{\bar{\sigma}}_1^6 - 5\bar{\bar{\sigma}}_1^4\bar{\bar{\sigma}}_2 + 6\bar{\bar{\sigma}}_1^2\bar{\bar{\sigma}}_2^2 - \bar{\bar{\sigma}}_2^3, \end{aligned} \tag{2.13}$$

and performing elementary operations with columns of determinant (2.9), we obtain

$$Det\left\| T_{ij}^{(m)} \right\|_{i,j=1}^{2} = K^2 \, Det\left\| m_{ij} \right\|_{i,j=1}^{8}, m = \overline{1,\infty}, \tag{2.14}$$

$$K = (\chi_1 - \chi_2)(\chi_1 - \chi_3)(\chi_1 - \chi_4)(\chi_2 - \chi_3)(\chi_2 - \chi_4)(\chi_3 - \chi_4)\,. \tag{2.15}$$

The expressions for m_{ij} are given in Appendix 1. The equations (2.9) are equivalent to the equations

$$Det\left\|m_{ij}\right\|_{i,j=1}^{8} = 0\,, m = \overline{1,\infty}\,. \tag{2.16}$$

By taking into account the possible relations between λ_1, λ_2 and λ_3 we conclude that equations (2.16) determine frequencies of the corresponding types of vibrations. For $\lambda_1 = \lambda_2 = \lambda_3 = \lambda,$ the equations (2.4) are the characteristic equations of the system (1.1), and the equations (2.16) are the dispersion equations of the problem (1.1)-(1.4).

In Section 5, the asymptotics of the dispersion equations (2.16) for $\varepsilon_m = 1/\theta_m R \to 0$ (transition to a cantilever rectangular plate or to vibrations localized at the free edges of the cylindrical panel with rigid – clamped edge generator) and for $\theta_m s \to \infty$ (transition to a semi-infinite cylindrical panel with free edges or to vibrations localized at the free edges of the cylindrical panel with rigid – clamped edge generator) are investigated. For checking the reliability of the asymptotic relations found in Section 5, the free planar and bending vibrations of a cantilever rectangular plate are investigated in the next two sections.

PLANAR VIBRATIONS OF AN ORTHOTROPIC CANTILEVER RECTANGULAR PLATE

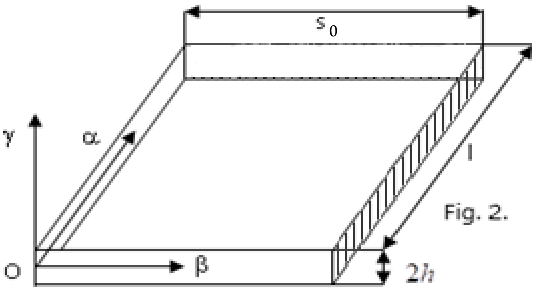


Fig. 2.

Let an orthotropic rectangular plate is defined in a triorthogonal system of rectilinear coordinates (α, β, γ) with the origin on the free face plane such that the coordinate plane $\alpha\beta$ coincides with the midsurface of the plate and the principal axes of symmetry of the material are aligned with the coordinate lines (Fig. 2). Let s *and* l be the width and the length of the plate, respectively. The problem of the existence of free planar vibrations of a cantileverrectangular plate is investigated. As the initial equations consider the equations of low-amplitude planar vibrations of the classical theory of orthotropic plates (Ambartsumyan S.A. 1974)

$$-B_{11}\frac{\partial^2 u_1}{\partial\alpha^2}-B_{66}\frac{\partial^2 u_1}{\partial\beta^2}-(B_{12}+B_{66})\frac{\partial^2 u_2}{\partial\alpha\partial\beta}=\lambda u_1;$$
$$-(B_{12}+B_{66})\frac{\partial^2 u_1}{\partial\alpha\partial\beta}-B_{66}\frac{\partial^2 u_2}{\partial\alpha^2}-B_{22}\frac{\partial^2 u_2}{\partial\beta^2}=\lambda u_2, \tag{3.1}$$

Here $\alpha\,(0\le\alpha\le l)$ and $\beta\,(0<\beta<s)$ are the orthogonal rectilinear coordinates of a point on the middle plane ; u_1 and u_2 are the displacements in α and β directions, respectively; $B_{ik},\ i,k=1,2,6$, are the coefficients of elasticity; $\lambda=\omega^2\rho$, where ωis the natural frequency; and ρ is the density of the material. The boundary conditions have the form

$$\left.\frac{\partial u_1}{\partial\alpha}+\frac{B_{12}}{B_{11}}\frac{\partial u_2}{\partial\beta}\right|_{\alpha=0,l}=\left.\frac{\partial u_2}{\partial\alpha}+\frac{\partial u_1}{\partial\beta}\right|_{\alpha=0,l}=0, \tag{3.2}$$

$$\left.\frac{B_{12}}{B_{11}}\frac{\partial u_1}{\partial\alpha}+\frac{\partial u_2}{\partial\beta}\right|_{\beta=0}=\left.\frac{\partial u_2}{\partial\alpha}+\frac{\partial u_1}{\partial\beta}\right|_{\beta=0}=0\,. \tag{3.3}$$

$$u_1|_{\beta=s_0}=u_2|_{\beta=s_0}=0. \tag{3.4}$$

Here conditions (3.2) and (3.3) mean that the edges $\alpha=0,l$ and $\beta=0$ are free, while conditions (3,4) indicate that the edge $\beta=s_0$ is rigid-clamped. The problem (3.1)-(3.4) does not allow separation of variables. The differential operator corresponding to this problem is self-conjugate and nonnegative definite. Therefore, the generalized Kantorovich-Vlasov method of the reduction to ordinary differential equations can be used to find vibration eigenfrequencies and eigenmodes (Vlasov V.Z. (1932))-(Mikhlin S.G. 1970). The solution of the system (3.1) is searched in the form

$$(u_1,u_2)=\{u_m w_m'(\theta_m\alpha),\ v_m w_m(\theta_m\alpha)\}\exp(\theta_m y\beta),\quad m=\overline{1,+\infty}. \tag{3.5}$$

In this case, the conditions (3.2) are satisfied automatically. Let us insert (3.5) into Eq. (3.1). Then, the obtained equations are multiplied by vector functions $\left(w_m'(\theta_m\alpha),w_m(\theta_m\alpha)\right)$ in a scalar way and integrated in the limits from 0 to l. As a result, the system of equations is obtained

$$\left(\beta''_m - \frac{B_{66}}{B_{11}}(y^2+\eta_m^2)\right)u_m - \frac{B_{12}+B_{66}}{B_{11}} y v_m = 0,$$

$$\frac{B_{12}+B_{66}}{B_{22}}\beta'_m\, y u_m - \left(y^2 - \frac{B_{66}}{B_{22}}\left(\beta'_m - \eta_m^2\right)\right)v_m = 0, \tag{3.6}$$

where $\eta_m^2 = \lambda/\left(\theta_m^2 B_{66}\right)$, θ_m and β'_m, β''_m are determined in Eq. (2) and (4), respectively. By equating the determinant of system (3.6) to zero, the following characteristic equations of the system of equations (3.1) are found:

$$c_m = \frac{B_{22}}{B_{11}} y^4 - B_2 y^2 + \frac{B_{22}+B_{66}}{B_{11}}\eta_m^2 y^2 + (\beta'_m - \eta_m^2)\left(\beta''_m - \frac{B_{66}}{B_{11}}\eta_m^2\right) = 0,\ m = \overline{1,+\infty}\,. \tag{3.7}$$

Let y_1 and y_2 be various roots of Eq. (3.7) with non-positive real parts and $y_{2+j} = -y_j$, $j = 1,2$. As the solution of system (3.6) for $y = y_j$, $j = \overline{1,4}$, we take

$$u_m^{(j)} = y_j^2 - \frac{B_{66}}{B_{22}}(\beta'_m - \eta_m^2),\ \ v_m^{(j)} = \frac{B_{12}+B_{66}}{B_{22}}\beta' y_j,\ j = \overline{1,4}\,. \tag{3.8}$$

The solution of the problem (3.1) - (3.4) can be presented in the form

$$\left(u_1, u_2\right) = \left(\sum_{j=1}^4 u_m^{(j)} w'_m(\theta_m \alpha)\exp(\theta_m y_j \beta) w_j, \sum_{j=1}^4 v_m^{(j)} w_m(\theta_m \alpha)\exp(\theta_m y_j \beta) w_j\right) \tag{3.9}$$

Let us insert Eq. (3.9) into the boundary conditions (3.3) and (3.4). Each of the obtained equation is multiplied by $w(\theta_m \alpha)$ or by $w'(\theta_m \alpha)$, and then integrated in the limits from 0 to l. As a result, the systems of equations are obtained

$$\begin{cases} \sum_{j=1}^4 R_{1j}^{(m)} w_j = 0, & \sum_{j=1}^4 R_{5j}^{(m)} \exp(z_j) w_j = 0, \\ \sum_{j=1}^4 R_{2j}^{(m)} w_j = 0, & \sum_{j=1}^4 R_{6j}^{(m)} \exp(z_j) w_j = 0. \end{cases} \quad m = \overline{1,+\infty}\,. \tag{3.10}$$

$$R_{1j}^{(m)} = y_j^2 + \frac{B_{12}}{B_{22}}(\beta'_m - \eta_m^2),\ \ R_{2j}^{(m)} = y_j\left(y_j^2 + \frac{B_{12}}{B_{22}}\beta'_m + \frac{B_{66}}{B_{22}}\eta_m^2\right),$$

$$R_{5j}^{(m)} = y_j^2 - \frac{B_{66}}{B_{22}}(\beta'_m - \eta_m^2),\ R_{6j}^{(m)} = \frac{B_{12}+B_{66}}{B_{22}}\beta'_m y_j,\ z_j = \theta_m y_j s,\ j = \overline{1,4}. \tag{3.11}$$

By equating the determinant Δ_e of the system (3.10) to zero and performing elementary operations with columns of the determinant the following dispersion equation is obtained

$$\Delta_e = \exp(-z_1 - z_2)(y_2 - y_1)^2\, Det\left\|l_{ij}\right\|_{i,j}^4 = 0\ . \tag{3.12}$$

$$l_{11} = R_{11}^{(m)},\ l_{12} = y_1 + y_2,\ l_{13} = l_{11}\exp(z_1),\ l_{14} = l_{12}\exp(z_2) + l_{11}[z_1 z_2];$$

$$l_{21}=R_{21}^{(m)},\ l_{22}=y_1y_2+\left(B_{11}B_{22}\beta_m''-B_{12}^2\beta_m'-B_{12}B_{66}\beta_m'\right)/(B_{22}B_{66})-\eta_m^2,$$
$$l_{23}=-l_{21}\exp(z_1),\ l_{24}=-l_{22}\exp(z_2)-l_{21}[z_1z_2];$$
$$n_{31}=R_{51}^{(m)}=y_j^2-\frac{B_{66}}{B_{22}}(\beta_m'-\eta_m^2),\ n_{32}=y_1+y_2,$$
$$l_{31}=n_{31}\exp(z_1),\ l_{32}=n_{32}\exp(z_2)+n_{31}[z_1z_2];l_{33}=n_{31},\ l_{34}=n_{32}, \tag{3.13}$$
$$n_{41}=R_{61}^{(m)}=\frac{B_{12}+B_{66}}{B_{22}}y_1\beta_m',\ n_{42}=\frac{B_{12}+B_{66}}{B_{22}}\beta_m';$$
$$l_{41}=n_{41}\exp(z_1),\ l_{42}=n_{42}\exp(z_2)+n_{41}[z_1z_2];l_{43}=-n_{41},\ l_{44}=-n_{42},$$
$$z_j=\theta_my_js,\ [z_1z_2]=\theta_ms(\exp(z_2)-\exp(z_1))/(z_2-z_1).$$

The equation (3.12) is equivalent to the equation

$$\begin{gathered}Det\left|l_{ij}\right|_{i,j=1}^4=\left((B_{12}+B_{66})/B_{22}\right)^2K_{2m}(\eta_m^2)Q(\eta_m^2)(1+\beta_m'\exp(2(z_1+z_2)))-\\ 4(l_{21}l_{11}n_{32}n_{42}+l_{12}l_{22}n_{31}n_{41})\exp(z_1+z_2)+(l_{11}l_{22}+l_{21}l_{12})(n_{32}n_{41}+n_{31}n_{42})(\exp(2z_1)+\exp(2z_2))+\\ 2(l_{11}l_{21}(n_{31}n_{42}+n_{32}n_{41})+n_{31}n_{41}\left(l_{11}l_{22}+l_{21}l_{12}\right))(\exp(z_2)-\exp(z_1))[z_1z_2]+4l_{11}l_{21}n_{41}n_{31}[z_1z_2]^2=0.\end{gathered} \tag{3.14}$$

$$K_{2m}\left(\eta_m^{(2)}\right)=(\beta_m'-\eta_m^2)\left(\frac{B_{11}B_{22}\beta_m''-B_{12}^2\beta_m'}{B_{22}B_{66}}-\eta_m^2\right)-\eta_m^2y_1y_2,\ Q(\eta_m^2)=y_1y_2+\frac{B_{66}}{B_{22}}(\beta_m'-\eta_m^2).$$

If y_1 and y_2 are the roots of Eq.(3.7) with negative real parts, then, at $\theta_ms\to\infty$, the roots of Eq. (3.14) are approximated by the roots of the equation

$$K_{2m}\left(\eta_m^{(2)}\right)=(\beta_m'-\eta_m^2)\left(\frac{B_{11}B_{22}\beta_m''-B_{12}^2\beta_m'}{B_{22}B_{66}}-\eta_m^2\right)-\eta_m^2y_1y_2=0. \tag{3.16}$$

The equation (3.16) is an analogue of the Rayleigh equation for a long enough orthotropic rectangular plate with a free side (compare with (Gulgazaryan, G.R., Gulgazaryan L.G., Saakyan R.D. 2008) – (Ghulghazaryan G.R., Ghulghazaryan L.G. 2020). Thus, the eigenfrequencies of the problem (3.1) - (3.4) can be found from (3.14).

To find the corresponding eigenmodes, the coefficients $w_j,\ j=\overline{1,4}$ have to be determined from the system of equations (3.10) and inserted into (3.9). We can take

$$w_1=\frac{R_1^{(m)}\exp(z_1+2z_2)+R_{52}^{(m)}R_2^{(m)}\exp(z_1)-2R_{12}^{(m)}R_{22}^{(m)}R_{51}^{(m)}\exp(z_2)}{R_{51}^{(m)}R_1^{(m)}-R_{51}^{(m)}R_2^{(m)}\exp(2z_1)+2R_{11}^{(m)}R_{21}^{(m)}R_{52}^{(m)}\exp(z_1+z_2)},$$
$$w_2=\frac{R_{51}^{(m)}R_2^{(m)}\exp(z_2)-R_{51}^{(m)}R_1^{(m)}\exp(2z_1+z_2)-2R_{11}^{(m)}R_{21}^{(m)}R_{52}^{(m)}\exp(z_1)}{R_{51}^{(m)}R_1^{(m)}-R_{51}^{(m)}R_2^{(m)}\exp(2z_1)+2R_{11}^{(m)}R_{21}^{(m)}R_{52}^{(m)}\exp(z_1+z_2)}, \tag{3.17}$$
$$w_3=-\frac{R_{52}^{(m)}R_1^{(m)}\exp(z_1)+R_{52}^{(m)}R_2^{(m)}\exp(z_1+2z_2)-2R_{12}^{(m)}R_{22}^{(m)}R_{51}^{(m)}\exp(2z_1+z_2)}{R_{51}^{(m)}R_1^{(m)}-R_{51}^{(m)}R_2^{(m)}\exp(2z_1)+2R_{11}^{(m)}R_{21}^{(m)}R_{52}^{(m)}\exp(z_1+z_2)},$$
$$w_4=\exp(z_2),\ R_1^{(m)}=R_{11}^{(m)}R_{22}^{(m)}-R_{12}^{(m)}R_{21}^{(m)},\ R_2^{(m)}=R_{11}^{(m)}R_{22}^{(m)}+R_{12}^{(m)}R_{21}^{(m)}.$$

as solutions to the system of equations (3.10) at a given dimensionless eigenfrequency characteristic η_m.

BENDING VIBRATIONS OF AN ORTHOTROPIC CANTILEVER RECTANGULAR PLATE

Consider an orthotropic rectangular plate with thickness h, width s, and length l (Fig. 2). Consider now the problem of the existence of free bending vibrations of acantilever rectangular plate. Let us start with the equation of low-amplitude bending vibrations of the classical theory of orthotropic plates (**Ambartsumyan S.A.** 1974)

$$\mu^4\left(B_{11}\frac{\partial^4 u_3}{\partial\alpha^4}+2(B_{12}+2B_{66})\frac{\partial^4 u_3}{\partial\alpha^2\partial\beta^2}+B_{22}\frac{\partial^4 u_3}{\partial\beta^4}\right)=\lambda u_3\ , \tag{4.1}$$

where $\alpha\,(0\le\alpha\le l)$ and $\beta\,(0\le\beta\le s)$ are the orthogonal rectilinear coordinates of a point of the median plane of the plate; u_3 is the normal component of the displacement vector of a point of the median plane; $B_{ik},\ i,k=1,2,6$ are the elasticity coefficients; $\mu^4=h^2/12$; $\lambda=\omega^2\rho$, where ω is the natural frequency; ρ is the density of the material.

The boundary conditions are given as follows:

$$\left.\frac{\partial^2 u_3}{\partial\alpha^2}+\frac{B_{12}}{B_{11}}\frac{\partial^2 u_3}{\partial\beta^2}\right|_{\alpha=0,l}=\left.\frac{\partial^3 u_3}{\partial\alpha^3}+\frac{B_{12}+4B_{66}}{B_{11}}\frac{\partial^3 u_3}{\partial\alpha\partial\beta^2}\right|_{\alpha=0,l}=0\,, \tag{4.2}$$

$$\left.\frac{B_{12}}{B_{22}}\frac{\partial^2 u_3}{\partial\alpha^2}+\frac{\partial^2 u_3}{\partial\beta^2}\right|_{\beta=0}=\left.\frac{\partial^3 u_3}{\partial\beta^3}+\frac{B_{12}+4B_{66}}{B_{22}}\frac{\partial^3 u_3}{\partial\beta\partial\alpha^2}\right|_{\beta=0}=0\ , \tag{4.3}$$

$$u_3\big|_{\beta=s_0}=\left.\frac{\partial u_3}{\partial\alpha}\right|_{\beta=s_0}=0. \tag{4.4}$$

Here the conditions (4.2) and (4.3) mean that the edges $\alpha=0,l$ and $\beta=0$ are free; while the conditions (4,4) indicate that the edge generator $\beta=s_0$ is rigid-clamped. The problem (4.1)-(4.4) does not allow separation of variables. The differential operator corresponding to this problem is self-conjugate and nonnegative definite. Therefore, the generalized Kantorovich-Vlasov method of the reduction to ordinary differential equations can be used to find the vibration eigenfrequencies and eigenmodes (Vlasov V.Z. 1932) -))-(Mikhlin S.G. 1970). The solution of the system (4.1) is searched in the form

$$u_3 = w_m(\theta_m \alpha)\exp(\theta_m y\beta), \quad m = \overline{1,+\infty} , \tag{4.5}$$

where $w_m(\theta_m \alpha)$ is defined in (2). The conditions (4.2) are satisfied automatically. Substitute (4.5) into Eq.(4.1). After multiplying the resulting equation by $w_m(\theta_m \alpha)$ and integrating it in the limits from 0 to l the characteristic equations are obtained

$$R_{mm} = a^2\left(y^4 - \frac{2(B_{12}+2B_{66})}{B_{22}}\beta'_m y^2 + \frac{B_{11}}{B_{22}}\beta'_m \beta''_m \right) - \frac{B_{66}}{B_{22}}\eta_m^2 = 0, \; m = \overline{1,+\infty} , \tag{4.6}$$

$$\eta_m^2 = \frac{\lambda}{\theta_m^2 B_{66}}, \; a^2 = \theta_m^2 h^2 / 12 , \tag{4.7}$$

where θ_m and β'_m, β''_m are defined in Eqs. (2) and (4), respectively. Let y_3 and y_4 be various roots of Eq. (4.6) with non-positive real parts, $y_{2+j} = -y_j$, $j = 3,4$. The solution of the problem (4.1)- (4.4) is searched in the form

$$u_3 = \sum_{j=3}^{6} w_m(\theta_m \alpha)\exp(\theta_m y_j \beta) \, w_j . \tag{4.8}$$

By inserting Eq. (4.8) into the boundary conditions (4.3) and (4,4), and after multiplying the resulting equations by $w_m(\theta_m \alpha)$, and integrating them in the limits from 0 to l, the system of equations is obtained

$$\begin{cases} \sum_{j=3}^{6} R_{3j}^{(m)} W_j = 0, & \sum_{j=3}^{6} R_{4j}^{(m)} W_j = 0, \\ \sum_{j=3}^{6} R_{7j}^{(m)} W_j = 0, & \sum_{j=3}^{6} R_{8j}^{(m)} W_j = 0 . \end{cases} \tag{4.9}$$

$$\begin{gathered} R_{3j}^{(m)} = y_j^2 - \frac{B_{12}}{B_{22}}\beta'_m, \; R_{4j}^{(m)} = y_j^3 - \frac{B_{12}+4B_{66}}{B_{22}}\beta'_m y_j, \\ R_{7j}^{(m)} = \exp(z_j), \;\; R_{8j}^{(m)} = y_j \exp(z_j) \, ; \, z_j = \theta_m y_j s, \, j = \overline{3,6} . \end{gathered} \tag{4.10}$$

By equating the determinant of system (4.9) Δ_b to zero and performing elementary operations on the columns of the determinant, the dispersion equation is obtained

$$\Delta_b = \exp(-z_3 - z_4)(y_4 - y_3)^2 \, Det\|b_{ij}\|_{i,j}^{4} = 0 , \tag{4.11}$$

$$\begin{gathered} b_{11} = R_{33}^{(m)}, \; b_{12} = y_3 + y_4, \; b_{13} = b_{11}\exp(z_3), \; b_{14} = b_{12}\exp(z_4) + b_{11}[z_3 z_4]; \\ b_{21} = R_{43}^{(m)}, \; b_{22} = y_3 y_4 + \beta'_m B_{12}/B_{22}, \, b_{23} = -b_{21}\exp(z_3), \, b_{24} = -b_{22}\exp(z_4) - b_{21}[z_3 z_4]; \\ b_{31} = \exp(z_3), \; b_{32} = [z_3 z_4], \, b_{33} = 1, \, b_{34} = 0; \;\; b_{41} = y_3\exp(z_3), \; b_{42} = \exp(z_4) + y_3[z_3 z_4], \\ b_{43} = -y_3, \, b_{44} = -1; \; [z_3 z_4] = \theta_m s(\exp(z_4) - \exp(z_3))/(z_4 - z_3); \;\; z_j = \theta_m y_j s, \, j = 3,4. \end{gathered} \tag{4.12}$$

The equation (4.11) is equivalent to the equation

$$Det\left|b_{ij}\right|_{i,j=1}^{4} = -K_{1m}(\eta_m^2)\ (1+\exp(\ 2\ (z_3+z_4))) - 4y_3\, b_{12}\, b_{22}\exp(z_3+z_4) +$$
$$(b_{11}b_{22}+b_{21}b_{12})(\exp(2z_3)+\exp(2z_4)) + 2[b_{11}b_{21}+y_3(b_{11}b_{22}+b_{21}b_{12})(\exp(z_4)-\exp(z_3))[z_3z_4] + \tag{4.13}$$
$$4y_3b_{11}b_{21}[z_3z_4]^2 = 0,\ m=\overline{1,\infty}.$$

$$K_{1m}(\eta_m^2) = y_3^2y_4^2 + 4\frac{B_{66}}{B_{22}}\beta_m' y_3y_4 - \left(\frac{B_{12}}{B_{22}}\right)(\beta_m')^2. \tag{4.14}$$

If y_3 and y_4 are the roots of Eq.(4.6) with negative real parts, then, at $\theta_m s \to \infty$, the roots of Eq. (4.13) are approximated by the roots of the equation

$$K_{1m}(\eta_m^2) = y_3^2y_4^2 + 4\frac{B_{66}}{B_{22}}\beta_m' y_3y_4 - \left(\frac{B_{12}}{B_{22}}\right)^2(\beta_m')^2 = 0, m=\overline{1,\infty}\ . \tag{4.15}$$

The equation (4.15) is an analogue of the Konenkov equation for a long enough orthotropic rectangular plate with a free side (compare with (Gulgazaryan, G.R., Gulgazaryan R.G., Khachanyan A.A. 2013) – (Ghulghazaryan G.R., Ghulghazaryan L.G. 2020). Thus, eigenfrequencies of the problem (4.1)-(4.4) can be found from (4.13).

To find the corresponding eigenmodes, the coefficients $w_j,\ j=\overline{3,6}$ have to be determined from the system of equations (4.9) and inserted into Eqs. (4.8). As solutions to the system of equations (4.9) at a given dimensionless eigenfrequency characteristic η_m, it can be taken

$$W_3 = \frac{R_3^{(m)}\exp(z_3+2z_4) + R_4^{(m)}\exp(z_3) - 2R_{34}^{(m)}R_{44}^{(m)}\exp(z_4)}{R_3^{(m)} - R_4^{(m)}\exp(2z_3) + 2R_{33}^{(m)}R_{43}^{(m)}\exp(z_3+z_4)},$$

$$W_4 = \frac{R_4^{(m)}\exp(z_4) - R_3^{(m)}\exp(2z_3+z_4) - 2R_{33}^{(m)}R_{43}^{(m)}\exp(z_3)}{R_3^{(m)} - R_4^{(m)}\exp(2z_3) + 2R_{33}^{(m)}R_{43}^{(m)}\exp(z_3+z_4)}, \tag{4.16}$$

$$W_5 = -\frac{\exp(z_3)[R_3^{(m)} + R_4^{(m)}\exp(z_4) - 2R_{34}^{(m)}R_{44}^{(m)}\exp(z_3+z_4)]}{R_3^{(m)} - R_4^{(m)}\exp(2z_3) + 2R_{33}^{(m)}R_{43}^{(m)}\exp(z_3+z_4)},$$

$$W_6 = \exp(z_4);\ R_3^{(m)} = R_{33}^{(m)}R_{44}^{(m)} - R_{34}^{(m)}R_{43}^{(m)},\ R_4^{(m)} = R_{33}^{(m)}R_{44}^{(m)} + R_{34}^{(m)}R_{43}^{(m)}\ .$$

ASYMPTOTICS OF DISPERSION EQUATIONS (2.16)

1. *Asymptotics of Dispersion Equations (2.16) at* $\varepsilon_m \to 0$. Using the previous formulas, we assume that $\eta_{1m}=\eta_{2m}=\eta_{3m}=\eta_m$. Then, as $\varepsilon_m \to 0$, Eqs. (2.4) transform into

$$c_m = \frac{B_{22}}{B_{11}}y^4 - B_2y^2 + \frac{B_{22}+B_{66}}{B_{11}}\eta_m^2y^2 + (\beta_m' - \eta_m^2)\left(\beta_m'' - \frac{B_{66}}{B_{11}}\eta_m^2\right) = 0, m=\overline{1,+\infty}\ , \tag{5.1}$$

$$R_{mm}=a^2\left(y^4-\frac{2(B_{12}+2B_{66})}{B_{22}}\beta'_m y^2+\frac{B_{11}}{B_{22}}\beta'_m\beta''_m\right)-\frac{B_{66}}{B_{22}}\eta_m^2=0\,,m=\overline{1,+\infty}\,. \quad (5.2)$$

Here the limiting process $\varepsilon_m \to 0$ is understood in the sense that by fixing the radius R and $b=s_0$ – the distance between the boundary generatrices of the cylindrical panel, a transition to a cylindrical panel of radius $R'=nR$ and to the limit $\varepsilon'_m=1/(n\theta_m R)=\varepsilon_m/n\to 0$ at $n\to\infty$ is performed.

The equations (5.1) and (5.2) are characteristic equations for the equations of planar and bending vibrations of orthotropic cantilever plates, respectively. The roots of the Eqs. (5.1) and (5.2) with non-positive real parts, as in Sections 3 and 4, are denoted by y_1, y_2 and y_3, y_4, respectively. In the same way as in **(Gulgazaryan G.R.** (2004)), it is proved that for

$$\varepsilon_m<<1;\quad y_i\neq y_j,\ i\neq j, \quad (5.3)$$

the roots χ^2 of Eqs. (2.4) can be presented as

$$\chi_i^2=y_i^2+\alpha_i^{(m)}\varepsilon_m^2+\beta_i^{(m)}\varepsilon_m^4+\ldots,\ i=\overline{1,4},\ m=\overline{1,+\infty}\ . \quad (5.4)$$

Under the condition (5.3), considering the relations (2.8), (2.14) and (5.4) and the fact that

$$M_{3j}^{(m)}=M_{4j}^{(m)}=M_{7j}^{(m)}=M_{8j}^{(m)}=O(\varepsilon_m^2),\ j=1,2\ , \quad (5.5)$$

Eq. (2.16) can be reduced to the form

$$Det\left\|m_{ij}\right\|_{i,j=1}^{8}=N^2(\eta_m^2)K_{3m}^2(\eta_m^2)Det\left|l_{ij}\right|_{i,j=1}^{4}Det\left|b_{ij}\right|_{i,j=1}^{4}+O(\varepsilon_m^2)=0,\ m=\overline{1,+\infty}, \quad (5.6)$$

where $Det\left|l_{ij}\right|_{i,j=1}^{4}$ and $Det\left|b_{ij}\right|_{i,j=1}^{4}$ are determined by (3.14) and (4.13), respectively, and

$$N(\eta_m^2)=(y_3+y_1)(y_3+y_2)(y_4+y_1)(y_4+y_2),$$

$$K_{3m}(\eta_m^2)=\left\{(\beta'_m-\eta_m^2)\left(\frac{B_{11}}{B_{22}}\beta''_m-\frac{B_{66}}{B_{22}}\eta_m^2\right)\left(\frac{B_{22}}{B_{11}}+a^2\left(\frac{B_{66}}{B_{11}}\eta_m^2+\frac{B_{12}^2+3B_{12}B_{66}+4B_{66}^2}{B_{11}B_{66}}\beta'_m\right)\right)^2+\right.$$
$$\left(\frac{B_{11}B_{22}\beta''-B_{12}^2\beta'_m-2B_{12}B_{66}\beta'_m}{B_{22}B_{66}}-\frac{B_{22}+B_{66}}{B_{22}}\eta_m^2\right)\left(\frac{B_{22}}{B_{11}}+a^2\left(\frac{B_{66}}{B_{11}}\eta_m^2+\frac{B_{12}^2+3B_{12}B_{66}+4B_{66}^2}{B_{11}B_{66}}\beta'_m\right)\right)X$$
$$\left(\frac{B_{22}}{B_{11}}\eta_m^2-\frac{B_{11}B_{22}\beta''-B_{12}^2\beta'_m-B_{12}B_{66}\beta'_m}{B_{11}B_{66}}+a^2\left(\frac{B_{66}}{B_{11}}\eta_m^2-\beta''_m\right)\left(\eta_m^2+\frac{B_{12}+3B_{66}}{B_{66}}\beta'_m\right)\right)+ \quad (5.7)$$
$$\left.\left(\frac{B_{22}}{B_{11}}\eta_m^2-\frac{B_{11}B_{22}\beta''-B_{12}^2\beta'_m-B_{12}B_{66}\beta'_m}{B_{11}B_{66}}+a^2\left(\frac{B_{66}}{B_{11}}\eta_m^2-\beta''_m\right)\left(\eta_m^2+\frac{B_{12}+3B_{66}}{B_{66}}\beta'_m\right)\right)^2\right\}\left(\frac{B_{22}}{(B_{12}+B_{66})\beta'_m}\right)^2.$$

From Eq. (5.6), it follows that in the limit $\varepsilon_m \to 0$, Eqs. (2.16) decompose into the totality of equations

$$Det\left|l_{ij}\right|_{i,j=1}^{4} = 0, m = \overline{1,+\infty};\ Det\left|b_{ij}\right|_{i,j=1}^{4} = 0, m = \overline{1,+\infty};\ K_{3M}(\eta_m^2) = 0, m = \overline{1,+\infty}\,. \tag{5.8}$$

Here the first two equations are the dispersion equations of the planar and bending vibrations, respectively, as in the similar problems for an orthotropic cantilever rectangular plate.

The roots of the third equation correspond to planar vibrations of a cylindrical panel. The third equation appears as the result of using the equation of the corresponding classical theory of orthotropic cylindrical shells.

If y_1, y_2 and y_3, y_4 are the roots of the Eqs. (5.1) and (5.2), respectively, with negative real parts, then, at $\theta_m s \to \infty$, Eqs. (2.16) and (5.6) will be transformed into the equations

$$\begin{gathered} Det\left\|m_{ij}\right\|_{i,j=1}^{8} = \left((B_{12}+B_{66})/B_{22}\right)^2 N^2(\eta_m^2) Q(\eta_m^2) K_{1m}(\eta_m^2) K_{2m}(\eta_m^2) K_{3m}^2(\eta_m^2) + \\ O(\varepsilon_m^2) + \sum_{j=1}^{4} O(\exp(z_j)) = 0,\ m = \overline{1,+\infty}\,. \end{gathered} \tag{5.9}$$

From Eqs. (5.9), it follows that, for $\varepsilon_m \to 0$ and $\theta_m s \to \infty$, the roots of dispersion equations (2.16) are approximated by roots of the equations

$$K_{1m}(\eta_m^2) = 0, m = \overline{1,+\infty};\ K_{2m}(\eta_m^2) = 0, m = \overline{1,+\infty};\ K_{3m}(\eta_m^2) = 0, m = \overline{1,+\infty}\,. \tag{5.10}$$

The first two equations of (5.10) are the dispersion equations of the bending and planar vibrations of width enough orthotropic cantilever rectangular plate with free sides (see Eqs. (4.15) and (3.16)). Hence, for small ε_m and large $\theta_m s$, the approximate values of the roots of Eqs. (2.16) correspond to the roots of Eqs. (5.8) and (5.10) (compare Tables 1 and 2).

2. *Asymptotics of Dispersion Equations (2.16) at $\theta_m s \to \infty$***.** In the previous formulas it was assumed that the roots χ_1, χ_2, χ_3, and χ_4 (the roots of Eq. (2.4)) have negative real parts. Then Eqs. (2.16) can be reduced to the form

$$Det\left\|m_{ij}\right\|_{i,j=1}^{8} = Det\left\|m_{ij}\right\|_{i,j=1}^{4} . Det\left\|m_{ij}\right\|_{i=5,j=1}^{8,4} + \sum_{j=1}^{4} O(\exp(\theta_m \chi_j s)) = 0, m = \overline{1,+\infty}\,. \tag{5.11}$$

Hence, it follows that for $\theta_m s \to \infty$ the roots of Eqs. (2.16) are approximated by roots of the equations

$$Det\left\|m_{ij}\right\|_{i,j=1}^{4} = 0, m = \overline{1,+\infty};\ Det\left\|m_{ij}\right\|_{i=5,j=1}^{8,4} = 0,\ m = \overline{1,+\infty}\,. \tag{5.12}$$

The first totality of Eqs. (5.12) determines all possible localized free vibrations at the free edge generator of an orthotropic width enough cylindrical panel, or determines all possible localized free vibrations at the free edge generator of an orthotropic cylindrical panel with rigid- clamped edge generator.

The second totality of Eqs. (5.12) determines all possible localized free vibrations of an orthotropic width enough cylindrical panel with rigid- clamped edge generator.

Notice, that if $\varepsilon_m \to 0$, the equations of (5.12) have the following asymptotic forms

$$\begin{aligned} Det\left\|m_{ij}\right\|_{i,j=1}^{4} &= \left((B_{12}+B_{66})/B_{11}\right)^2 N(\eta_m^2)K_{1m}(\eta_m^2)K_{2m}(\eta_m^2)K_{3m}(\eta_m^2)+O(\varepsilon_m^2),\ m=\overline{1,+\infty}, \\ Det\left\|m_{ij}\right\|_{i=5,j=1}^{8,4} &= N(\eta_m^2)K_{3m}(\eta_m^2)Q(\eta_m^2)+O(\varepsilon_m^2),\ m=\overline{1,+\infty}\ . \end{aligned} \tag{5.13}$$

Thus, by taking into account (5.12) and (5.13), we conclude that the dispersion equations (2.16) for $\theta_m s \to \infty$ and $\varepsilon_m \to 0$ take the form (5.9).

NUMERICAL RESULTS

In the Table 1 the values of some η_m roots of the equations of (5.8) are given for an orthotropic cantilever rectangular boron plastic plate with parameters (Ghulghazaryan G.R, Lidskii V.B. (1982))

$$\begin{aligned} &\rho = 2\cdot 10^3\, kg/M^3;\ E_1 = 2.646.10^{11}\, N/M^2; \\ &E_2 = 1.323\cdot 10^{10};\ \ G = 9.604\cdot 10^9;\nu_1 = 0.2;\nu_2 = 0.01 \end{aligned} \tag{6.1}$$

and geometrical parameters $l = 4;\ s_0 = 5,\ s_0 = 15.$

Table 1. Roots of equations $K_{3m}(\eta_m^2) = 0$ and Characteristics of Eigenfrequencies of a cantilever rectangular Boron Plate

m	θ_m	$K_{3m}(\eta_m^2)=0;$	$Det\left\|b_{ij}\right\|_{i,j=1}^{4}=0, s=5;$ $Det\left\|b_{ij}\right\|_{i,j=1}^{4}=0, s=15.$	$Det\left\|l_{ij}\right\|_{i,j=1}^{4}=0, s=5;$ $Det\left\|l_{ij}\right\|_{i,j=1}^{4}=0, s=15.$
1	1.95473	4.25538	0.04886 0.04886	- -
2	2.74891	4.94711	0.08584	0.98541

			0.08584	0.98398
3	3.52957	4.81097	0.10928 0.10922	1.00260 1.00225
4	4.27693	4.75564	0.12794 0.12789	0.98026 0.98012
12	12.7680	4.78787	0.38733 0.38731	0.98696 0.98696
13	13.8782	4.78717	0.42100 0.42099	0.98696 0.98696
14	14.9887	4.78640	0.45469 0.46233	0.98696 0.98696
15	16.0962	4.78555	0.48828 0.48827	0.98696 0.98696
16	17.1935	4.78465	0.52157 0.52157	0.98696 0.98696

In the Table 2 somedimensionless characteristics of the eigenvalues η_m for predominantly bending, predominantly planar and nonsymmetrical vibrations of an orthotropic cantilever cylindrical boron plastic panel with the same mechanical characteristics and the geometrical parameters: $R = 40; l = 4;\ s = 5.00326,\ s = 15.0893$. are given.

In the Table 2 after the characteristics of eigenfrequencies the type of vibration is indicated: ``$b$``- predominantly bending, ``$e$``- predominantly planar, and ``$n$`` for new type of vibrations. The elasticity modules E_1 and E_2 correspond to the directions of generatrix and directrix, respectively.

Table 2. Characteristics of Eigenfrequencies for Predominantly Bending, Predominantly Planar and Nonsymmetrical Vibrations of an Cylindrical Boron Plastic Panel with $l = 4,\ s = 5.00326,\ s = 15.0893.$

m	θ_m	$\eta_{1m} = \eta_{2m} = 0,$ $\eta_{3m} = \eta_m;$ $s = 5.00326;$ $s = 15.0893;$	$\eta_{1m} = \eta_{2m} =$ $\eta_m, \eta_{3m} = 0;$ $s = 5.00326;$ $s = 15.0893;$	$\eta_{1m} = \eta_{2m} = \eta_{3m} = \eta_m,$ $s = 5.00326;$ $s = 15.0893;$
1	1.95391	0.05393 b 0.05127 b	- - 4.25538 n	0.05006 b - 0.05006 b - 4.25538 n
2	2.74776	0.08710 b 0.08638 b	0.98548 e 0.98425 e 4.94711 n	0.08629 b 0.98541 e 0.08588 b 0.98439 e 4.94711 n
3	3.52810	0.11012b 0.10948 b	1.00245 e 1.00126 e 4.81097 n	0.10949 b 1.00167 e 0.10887 b 1.00234 e 4.81097 n
4	4.27542	0.12807 b 0.12803 b	0.98005 e 0.97991 e 4.75564 n	0.12805 b 0.98026 e 0.12757 b 0.98012 e 4.75564 n
12	12.7679	038733 b 0.38732 b	0.98696 e 0.98696 e 4.78787 n	0.38733 b 0.98696 e 0.38606 b 0.98696 e 4.78787 n
13	13.8785	0.42116 b 0.42102 b	0.98696 e 0.98696 e 4.78717 n	0.42116 b 0.98696 e 0.42100 b 0.98696 e 4.78717 n
14	14.9864	0.45476 b 0.45461 b	0.98696 e 0.98696 e 4.78640 n	0.45476 b 0.98696 e 0.45318 b 0.98696 e 4.78640 n
15	16.1102	0.48884 b 0.48875 b	0.98696 e 0.98696 e 4.78555 n	0.48884 b 0.98696 e 0.48720 b 0.98696 e 4.78555 n
16	17.2065	0.52197 b 0.52196 b	0.98696 e 0.98696 e 4.78465 n	0.52197 b 0.98696 e 0.52038 b 0.98696 e 4.78465 n

In the Table 2, the case with $\eta_1 = \eta_2 = \eta_3 = \eta$ corresponds to the problem (1.1)-(1.4). The case with $\eta_1 = \eta_2 = 0$ and $\eta_3 = \eta$ corresponds to the problem (1.1)-(1.4), where are no tangential components of the inertia force, i.e., we have the predominantly bending type of vibrations. The case with $\eta_1 = \eta_2 = \eta, \eta_3 = 0$ corresponds to the predominantly planar type of vibrations.

The following equalities hold for isotropic materials:

$$\frac{B_{12}}{B_{11}} = \frac{B_{12}}{B_{22}} = \upsilon, \ \frac{B_{66}}{B_{11}} = \frac{B_{66}}{B_{22}} = \frac{1-\upsilon}{2}. \tag{6.2}$$

Hence, in the dispersion equations and the characteristics calculations it can be set $B_{11} = B_{22} = 1, \ B_{12} = \upsilon, \ B_{66} = (1-\upsilon)/2$.

CONCLUSION

Numerical calculations show that the first eigenfrequencies localized at the free generator of the cylindrical panel with rigid- clamped generator where the normal component of inertia force is not zero are the frequencies of the predominantly bending type. Along with the first frequencies of quasitransverse vibrations, there are frequencies of undamped quasitangential vibrations. With the increase of m, these vibrations become of Rayleigh type. The analysis of the numerical data indicates that for $\varepsilon_m \to 0$ free vibrations of cylindrical panel with rigid- clamped generator decompose into quasitransverse and quasitangential vibrations, and their frequencies tend to the frequencies of a cantilever rectangular plate. Numerical results show that asymptotic formulas (5.6) of dispersion equation (2.16) and the mechanism presented here are good reference points for finding the eigenfrequencies of the problem (1.1)-(1.4). The first eigenfrequencies of vibrations of cylindrical panel with rigid- clamped generator depend on the chosen basic functions satisfying the conditions of free edges at ends. For $\theta_m \to \infty$, the frequencies of vibrations at free generator of a finite cylindrical panel with rigid- clamped generator become practically independent of the basic functions and of the boundary conditions on other edges.

APPENDIX

The analytical expressions for m_{ij} are given below:

$$m_{11} = H\chi_1^6 + d_1\chi_1^4 + d_2\chi_1^2 + d_3; \;\; m_{12} = H\bar{\bar{f}}_5 + d_1\bar{\bar{f}}_3 + d_2\bar{\bar{f}}_1;$$
$$m_{13} = H\bar{f}_4 + d_1\bar{f}_2 + d_2; \;\; m_{14} = Hf_3 + d_1 f_1;$$

$$m_{21} = T\chi_1^5 + d_4\chi_1^3 + d_5\chi_1; \;\; m_{22} = T\bar{\bar{f}}_4 + d_4\bar{\bar{f}}_2 + d_5; \;\; m_{23} = T\bar{f}_3 + d_4\bar{f}_1; \;\; m_{24} = Tf_2 + d_4;$$

$$m_{31} = F\chi_1^6 + d_6\chi_1^4 + d_7\chi_1^2 + d_8; \;\; m_{32} = F\bar{\bar{f}}_5 + d_6\bar{\bar{f}}_3 + d_7\bar{\bar{f}}_1;$$

$$m_{33} = F\bar{f}_4 + d_6\bar{f}_2 + d_7; \;\; m_{34} = F\, f_3 + d_6 f_1;$$

$$m_{41} = F\chi_1^7 + d_9\chi_1^5 + d_{10}\chi_1^3 + d_{11}\chi_1; \;\; m_{42} = F\bar{\bar{f}}_6 + d_9\bar{\bar{f}}_4 + d_{10}\bar{\bar{f}}_2 + d_{11};$$

$$m_{43} = F\bar{f}_5 + d_9\bar{f}_3 + d_{10}\bar{f}_1; \;\; m_{44} = Ff_4 + d_9 f_2 + d_{10};$$

$$m_{i5} = (-1)^{i-1} m_{i1}\exp(z_1);\; m_{i6} = (-1)^{i-1}(m_{i2}\exp(z_2) + m_{i1}[z_1 z_2]);$$
$$m_{i7} = (-1)^{i-1}(m_{i3}\exp(z_3) + m_{i2}[z_2 z_3] + m_{i1}[z_1 z_2 z_3]); \qquad (7.1)$$
$$m_{i8} = (-1)^{i-1}(m_{i4}\exp(z_4) + m_{i3}[z_3 z_4] + m_{i2}[z_2 z_3 z_4] + m_{i1}[z_1 z_2 z_3 z_4]),\; i = \overline{1,4};$$

$$n_{51} = S\chi_1^4 + \gamma_1\chi_1^2 + \gamma_2, \;\; n_{52} = S\,\bar{\bar{f}}_3 + \gamma_1\bar{\bar{f}}_1, n_{53} = S\bar{f}_2 + \gamma_1, n_{54} = S\, f_1;$$

$$n_{61} = H\chi_1^5 + \gamma_3\chi_1^3 + \gamma_4\chi_1, \;\; n_{62} = H\bar{\bar{f}}_4 + \gamma_3\bar{\bar{f}}_2 + \gamma_4, n_{63} = H\bar{f}_3 + \gamma_3\bar{f}_1, n_{64} = Hf_4 + \gamma_3;$$

$$n_{71} = F(1 + a^2\varepsilon_m^2)\,\chi_1^4 + \gamma_5\chi_1^2 + \gamma_6, \;\; n_{72} = F(1 + a^2\varepsilon_m^2)\,\bar{\bar{f}}_3 + \gamma_5\bar{\bar{f}}_1,$$
$$n_{73} = F(1 + a^2\varepsilon_m^2)\,\bar{f}_2 + \gamma_5, n_{54} = F(1 + a^2\varepsilon_m^2)\, f_1;$$

$$n_{81} = F(1 + a^2\varepsilon_m^2)\,\chi_1^5 + \gamma_5\chi_1^3 + \gamma_6\chi_1, \; n_{82} = F(1 + a^2\varepsilon_m^2)\,\bar{\bar{f}}_4 + \gamma_5\bar{\bar{f}}_2 + \gamma_6;$$
$$n_{83} = F(1 + a^2\varepsilon_m^2)\,\bar{f}_3 + \gamma_5\bar{f}_1; \;\; m_{84} = F(1 + a^2\varepsilon_m^2)\, f_2 + \gamma_5;$$

$$m_{i1} = n_{i1}\exp(z_1);\; m_{i2} = n_{i2}\exp(z_2) + n_{i1}[z_1 z_2];$$
$$m_{i3} = n_{i3}\exp(z_3) + n_{i2}[z_2 z_3] + n_{i1}[z_1 z_2 z_3];$$
$$m_{i4} = n_{i4}\exp(z_4) + n_{i3}[z_3 z_4] + n_{i2}[z_2 z_3 z_4] + n_{i1}[z_1 z_2 z_3 z_4],\; i = \overline{5,8};$$

$$m_{5\;4+j} = n_{5\,j}, m_{6\;4+j} = -n_{6\,j}, m_{7\;4+j} = n_{7\,j}, m_{8\;4+j} = -n_{8\,j};$$

$$H = -a^2\,\frac{B_{22}}{B_{11}}; \quad T = \frac{B_{12}B_{22}}{B_{11}B_{66}}a^2;\; F = \frac{B_{22}}{B_{11}};\; S = \frac{(B_{12} + B_{66})B_{22}}{B_{11}B_{66}}a^2$$

$$d_1 = a^2\left(\frac{B_{11}B_{22}\beta'' - B_{12}^2\beta_m' + 4B_{66}^2\beta_m'}{B_{11}B_{66}} - \frac{B_{22}}{B_{11}}\left(\eta_{1m}^2 + \varepsilon_m^2\right)\right),$$

$$d_2 = -\frac{B_{66}}{B_{11}}\eta_{2m}^2 + a^2\beta_m'\,\frac{B_{12} + 4B_{66}}{B_{22}}\left(\frac{B_{22}}{B_{11}}\eta_{1m}^2 - B_1\right) - \varepsilon_m^2 a^2\left(\frac{B_{22}}{B_{11}}\eta_{1m}^2 - \frac{B_{11}B_{22}\beta_m'' - B_{12}^2\beta_m' + 4B_{66}^2\beta'}{B_{11}B_{22}}\right),$$

$$d_3 = -\left(\delta_m\beta_m' - \eta_{2m}^2\right)\left(\frac{B_{11}B_{22}\beta_m'' - B_{12}^2\beta_m'}{B_{11}B_{22}} - \frac{B_{66}}{B_{11}}\eta_{1m}^2\right),\;\; \delta_m = 1 + 4\varepsilon_m^2 a^2,$$

$$d_4 = a^2\left(\frac{B_{22}B_{12}}{B_{11}B_{66}}\varepsilon_m^2 + B_2 - \frac{2B_{12}}{B_{11}}\beta'_m - \frac{B_{22}}{B_{11}}\eta_{1m}^2\right),$$

$$d_5 = \frac{B_{22}}{B_{11}}\eta_{1m}^2 + \frac{B_{12}}{B_{11}}\eta_{2m}^2 - \frac{B_{11}B_{22}\beta''_m - B_{12}^2\beta'_m}{B_{11}B_{66}} + a^2\,\frac{B_{12}+4B_{66}}{B_{22}}\beta'_m\left(\frac{B_{22}}{B_{11}}\eta_{1m}^2 - \frac{B_{22}}{B_{66}}\beta''_m\right) - \frac{4B_{12}}{B_{11}}\beta'_m\varepsilon_m^2 a^2,$$

$$d_6 = \frac{B_{22}}{B_{11}}\eta_{1m}^2 + \frac{B_{66}}{B_{11}}\eta_{2m}^2 + \frac{B_{22}}{B_{11}}\varepsilon_m^2 - B_1,$$

$$d_7 = (\eta_{2m}^2 - \beta'_m)\left(\frac{B_{66}}{B_{11}}\eta_{1m}^2 - \beta''\right) + \frac{B_{12}}{B_{22}}\beta'_m\left(B_2 - \frac{B_{22}}{B_{11}}\eta_{1m}^2 - \frac{B_{66}}{B_{11}}\eta_{2m}^2\right) +$$

$$\varepsilon_m^2\left(4a^2\,\frac{B_{12}B_{66}}{B_{11}B_{22}}(\beta'_m)^2 - B_1 + \frac{B_{22}}{B_{11}}\eta_{1m}^2\right),$$

$$d_8 = -\frac{B_{12}}{B_{22}}\beta'_m\,(\eta_{2m}^2 - \beta'_m\delta_m)\left(\frac{B_{66}}{B_{11}}\eta_{1m}^2 - \beta''\right),$$

$$d_9 = \frac{B_{22}}{B_{11}}\eta_{1m}^2 + \frac{B_{66}}{B_{11}}\eta_{2m}^2 + \frac{B_{22}}{B_{11}}\varepsilon_m^2 - B_1 - \frac{4B_{66}}{B_{11}}\beta'_m,$$

$$d_{10} = (\eta_{2m}^2 - \beta'_m)\left(\frac{B_{66}}{B_{11}}\eta_{1m}^2 - \beta''\right) + \frac{B_{12}+4B_{66}}{B_{22}}\beta'_m\left(B_2 - \frac{B_{22}}{B_{11}}\eta_{1m}^2 - \frac{B_{66}}{B_{11}}\eta_{2m}^2\right) -$$

$$\varepsilon_m^2\left(B_1 + \frac{4B_{66}}{B_{11}}\beta' - \frac{B_{22}}{B_{11}}\eta_{1m}^2\right),$$

$$d_{11} = 4\varepsilon_m^2\beta'_m\left(\frac{B_{66}}{B_{22}}B_1 - \frac{B_{66}}{B_{11}}\eta_{1m}^2\right) - \frac{B_{12}+4B_{66}}{B_{22}}\beta'_m(\eta_{2m}^2 - \beta'_m)\left(\frac{B_{66}}{B_{11}}\eta_{1m}^2 - \beta''\right);$$

$$\gamma_1 = \frac{B_{12}B_{22}}{B_{11}B_{66}}a^2\varepsilon_m^2 - \frac{B_{22}}{B_{11}} - a^2\,\frac{(B_{12}+B_{66})(B_{12}+4B_{66})}{B_{11}B_{66}}\beta'_m,$$

$$\gamma_2 = -\frac{B_{22}}{B_{11}}\beta'_m(\delta_m - \eta_{2m}^2);\;\; \gamma_3 = \frac{B_{22}}{B_{11}} + a^2\left(\frac{B_{11}B_{22}\beta'' + B_{12}B_{66}\beta'_m + 4B_{66}^2\beta'_m}{B_{11}B_{66}} - \frac{B_{22}}{B_{11}}\eta_{1m}^2\right),$$

$$\gamma_4 = a^2\,\frac{B_{12}+4B_{66}}{B_{22}}\beta'_m\left(\frac{B_{22}}{B_{11}}\eta_{1m}^2 - \frac{B_{22}}{B_{66}}\beta''_m\right) + \frac{B_{22}}{B_{11}}\eta_{1m}^2 - B_1,$$

$$\gamma_5 = \frac{B_{22}}{B_{11}}\left(1+\varepsilon_m^2 a^2\right)\eta_{1m}^2 + \frac{B_{66}}{B_{11}}\eta_{2m}^2 - B_2 - a^2\varepsilon_m^2\,\frac{B_{11}B_{22}\beta'' + 4B_{66}^2\beta'_m}{B_{11}B_{66}},$$

$$\gamma_6 = (\eta_{2m}^2 - \beta'_m)\left(\frac{B_{66}}{B_{11}}\eta_{1m}^2 - \beta''\right) - 4a^2\varepsilon_m^2\,\frac{B_{66}}{B_{22}}\beta'_m\left(\frac{B_{22}}{B_{11}}\eta_{1m}^2 - \beta''_m\right).$$

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РЕЗЮМЕ

СВОБОДНЫЕ КОЛЕБАНИЯ ТОНКОЙ УПРУГОЙ ОРТОТРОПНОЙ ЦИЛИНДРИЧЕСКОЙ ПАНЕЛИ С ЖЕСТКО-ЗАЩЕМЛЕННОЙ ГРАНИЧНОЙ ОБРАЗУЮЩЕЙ

ГУЛГАЗАРЯН ГУРГЕН, ГУЛГАЗАРЯН ЛУСИНЕ

Используя систему уравнений соответствующей классической теории ортотропных цилиндрических оболочек, исследуются свободные колебания ортотропной тонкой упругой цилиндрической панели с жестко-защемленной граничной образующий. Для расчета собственных частот и идентификации соответствующих собственных мод используется обобщенный метод сведения к обыкновенным дифференциальным уравнениям Канторовича-Власова.

Получены дисперсионные уравнения для нахождения собственных частот возможных типов колебаний.

Установлена асимптотическая связь между дисперсионными уравнениями рассматриваемой задачи и аналогичной задачи для ортотропной прямоугольной консольной пластины. Приводится механизм, с помощью которого расчленяются возможные типы краевых колебаний.

На примере ортотропной цилиндрической панели получены приближенные значения безразмерной характеристики собственных частот колебаний.

Ключевые слова: *Свободные колебания, собственные частоты, цилиндрический панель, собственные функции.*

ԱՄՓՈՓՈՒՄ

ԵԶՐԻԾՆԻՉՈՎ ԿՈՇՏ ԱՄՐԱԿՑՎԱԾ ԲԱՐԱԿ ԱՌԱՁԳԱԿԱՆ ՕՐԹՈՏՐՈՊ ԳԼԱՆԱՁԵՎ ՎԱՀԱՆԱԿԻ ԱԶԱՏ ՏԱՏԱՆՈՒՄՆԵՐԸ

ՂՈՒԼՂԱԶԱՐՅԱՆ ԳՈՒՐԳԵՆ, ՂՈՒԼՂԱԶԱՐՅԱՆ ԼՈՒՍԻՆԵ

Օգտվելով օրթոտրոպ գլանային թաղանթների դասական տեսությանը համապատասխան հավասարումների համակարգից՝ հետազոտվում է եզրիծնիչով կոշտ ամրակցված բարակ առաձգական օրթոտրոպ գլանաձև վահանակի ազատ տատանումները: Սեփական հաճախությունների արժեքները և նրանց համապատասխան սեփական ֆունկցիաները գտնելու համար կիրառվում է սովորական դիֆերենցիալ հավասարումների բերման Կանտորովիչ-Վլասովի մեթոդը: Հնարավոր տիպերի սեփական տատանումների հաճախությունները գտնելու համար արտածված են դիսպերսիոն հավասարումներ:

Ասիմպտոտիկ կապ է հաստատված դիտարկվող խնդրի և ուղղանկյուն օրթոտրոպ բարձակային սալիդիսպերսիոն հավասարումների միջև: Բերված է մեխանիզմ, որի օգնությամբ տարանջատվում են հնարավոր տիպի եզրային տատանումները:

Օրթոտրոպ գլանային վահանակի օրինակի վրա ստացված են սեփական տատանման մոտավոր հաճախությունների անչափողական բնութագրիչները:

Բանալի բառեր՝ ազատ տատանումներ, սեփական հաճախություններ, գլանային վահանակ, սեփական ֆունկցիա:

Հոդվածը ստացվել է՝ 15.04.2023

Հոդվածն ուղարկվել է գրախոսման՝ 20.04.2023

Հոդվածը երաշխավորվել է տպագրության՝ 12.06.2023